

St George Girls High School

Trial Higher School Certificate Examination

2013



Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Begin each question in a new booklet
- Write your student number on each booklet.
- Board-approved calculators may be used.
- A table of standard integrals is provided at the back of this paper.
- Show all necessary working in Questions 11 – 16.
- Diagrams are not to scale.
- The mark allocated for each question is listed at the side of the question.

Total Marks – 100

Section I – Pages 2 – 4

10 marks

- Attempt Questions 1 – 10 using the answer sheet provided at the end of the paper
- Allow about 15 minutes for this section

Section II – Pages 5 – 10

90 marks

- Attempt Questions 11 – 16
- Allow about 2 hours 45 minutes for this section

Students are advised that this is a Trial Examination only and does not necessarily reflect the content or format of the Higher School Certificate Examination.

Section I

10 marks

Attempt Questions 1 to 10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

1. If the line $4x - ky = 6$ passes through the point $(-1, -2)$. The value of k is:

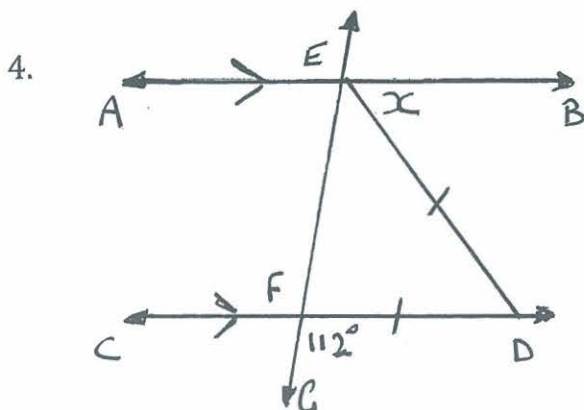
(A) -1
(B) 1
(C) -5
(D) 5

2. A parabola with equation $(x - 2)^2 = 8(y + 4)$ has its focus at the point:

(A) $(2, -4)$
(B) $(2, -2)$
(C) $(4, -4)$
(D) $(4, -2)$

3. From a block of clay exactly 10 statues can be made. If the linear dimensions of the statues are all halved then the number of smaller statues that can be made is:

(A) 10
(B) 20
(C) 40
(D) 80



If $AB \parallel CD$, $ED = FD$
and $\angle DFG = 112^\circ$
then $\angle BED =$

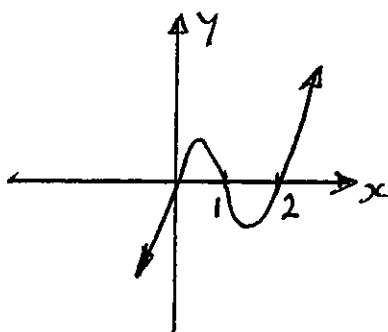
(A) 44° (B) 68° C. 24° D. 112°

Section I (cont'd)

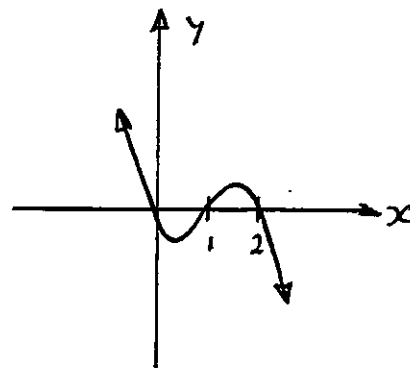
Marks

5. Which graph best illustrates $y = x(x - 1)(2 - x)$

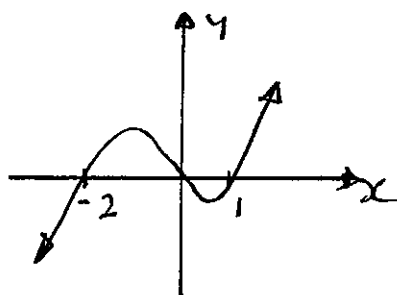
(A)



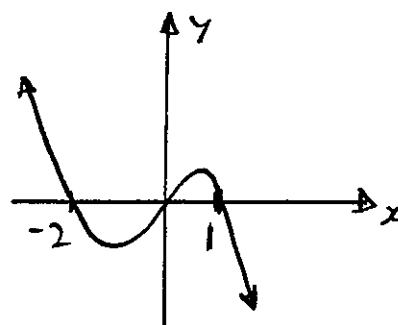
(B)



(C)



(D)



6. $y = 4\sin\frac{1}{2}x$ has amplitude and period of

(A) $4, \frac{1}{2}$

(B) $4, 2\pi$

(C) $\frac{1}{2}, 4$

(D) $4, 4\pi$

7. A particle moves according to the rule $x = \frac{1}{2}t^2 - 4t + c$ where x is the displacement from the origin after t seconds. Initially the particle is 8 metres from the origin. When the particle is at rest its displacement from the origin is:

(A) 0 metres

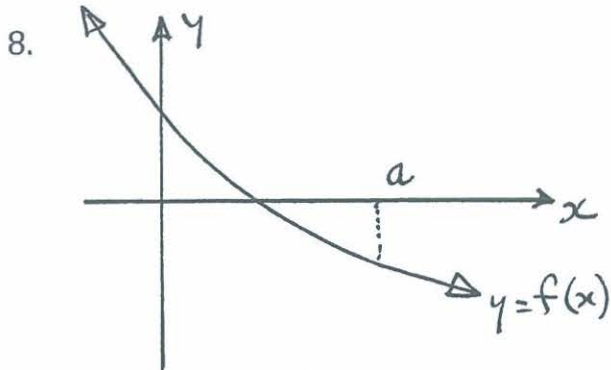
(B) 4 metres

(C) 8 metres

(D) 16 metres

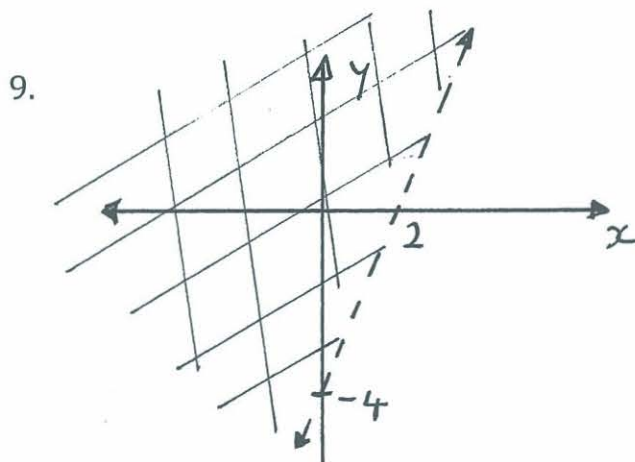
Section I (cont'd)

Marks



Which of the following is true at $x = a$

- (A) $f'(a) > 0$ and $f''(a) < 0$
- (B) $f'(a) > 0$ and $f''(a) > 0$
- (C) $f'(a) < 0$ and $f''(a) < 0$
- (D) $f'(a) < 0$ and $f''(a) > 0$



The shaded region is best described by the inequality.

- (A) $2x - y - 4 < 0$
- (B) $2x - y - 4 > 0$
- (C) $2x - y - 4 \leq 0$
- (D) $2x - y - 4 \geq 0$

10. The perpendicular distance from the line $3x - y = 4$ and the point $(2, 1)$ is given by:

- (A) $\frac{9}{\sqrt{5}}$
- (B) $\frac{9}{\sqrt{10}}$
- (C) $\frac{1}{\sqrt{5}}$
- (D) $\frac{1}{\sqrt{10}}$

Section II

90 Marks

Attempt Questions 11 – 16

All about 2 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

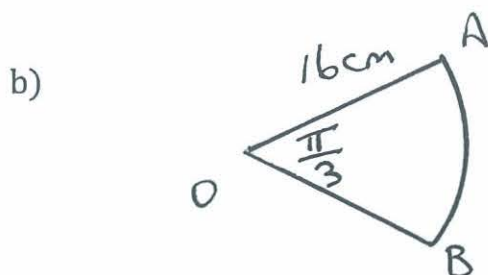
Question 11 – Start A New Booklet – (15 marks)	Marks
a) Evaluate $\frac{1.9^3 - 18}{\sqrt{2.1^4 - 0.8^3}}$ to 2 decimal places.	2
b) Solve $ 4 - 2x \leq 6$	2
c) Express as a single fraction with a rational denominator $\frac{1}{2 - \sqrt{3}} + \frac{1}{2\sqrt{2} + 3}$	3
d) Differentiate $x^3 \ln x$	2
e) Solve $\tan \theta = \sqrt{3}$ for $0 \leq \theta \leq 2\pi$	2
f) Factorise $125 - 8p^3$	2
g) Find the primitive of $\frac{6 - 3x^2}{x^2}$	2

Question 12 – Start A New Booklet – (15 marks)

Marks

- a) Evaluate $e^{3.7}$ correct to 4 significant figures.

2



AB is an arc of the circle centre O .

2

$OA = 16$ cm and $\angle AOB = \frac{\pi}{3}$.

Find the exact area of the sector AOB

- c) If

2

$$f(x) = \begin{cases} x + 2 & \text{for } x \leq -2 \\ 4 - x^2 & \text{for } -2 < x < 2 \\ 3x - 6 & \text{for } x \geq 2 \end{cases}$$

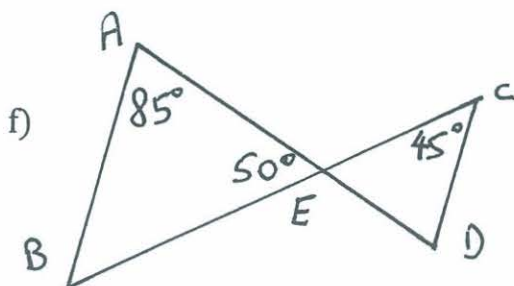
Evaluate $f(3) + f(0) - f(-2)$

- d) Solve $3^{2x} - 6(3^x) - 27 = 0$

2

- e) The first term of an arithmetic series is 5 and the 10th term is 4 times the second. Find the common difference.

2



Prove $AB \parallel CD$, stating all reasons.

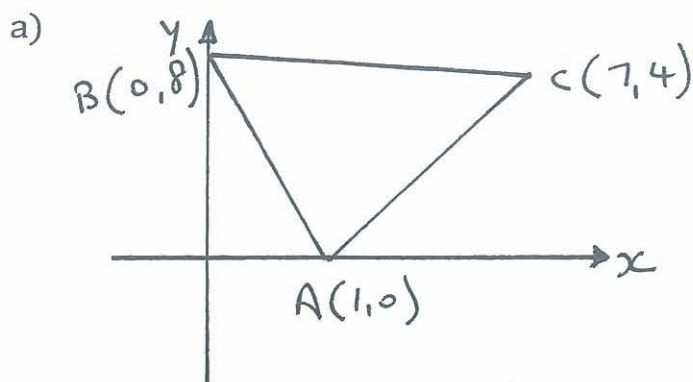
2

- g) Find $\frac{d}{dx} \log_e(\cos x)$, and hence find $\int \tan x \, dx$

3

Question 13 – Start A New Booklet – (15 marks)

Marks

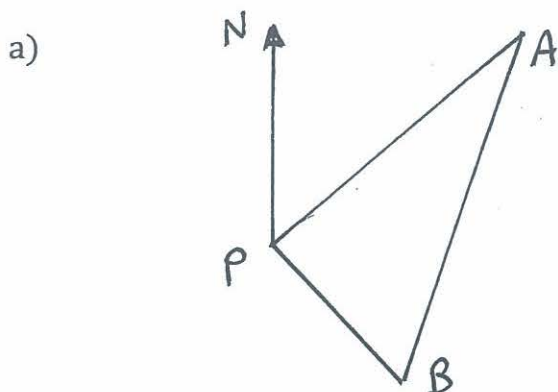


The points A , B and C have co-ordinates $(1, 0)$, $(0, 8)$ and $(7, 4)$ as shown on the diagram. The angle between CA and the positive x -axis is θ° .

- | | |
|--|---|
| (i) Find the gradient of CA | 1 |
| (ii) Calculate the size of θ , to the nearest degree. | 1 |
| (iii) Find the equation of CA | 1 |
| (iv) Find the coordinates of D , the midpoint of CD . | 1 |
| (v) Show $CA \perp BD$ | 2 |
| (vi) Calculate the area of $\triangle ABC$ | 2 |
| | |
| b) Prove $2 \cos^2 \theta + 1 = 3 - 2 \sin^2 \theta$ | 2 |
| | |
| c) Differentiate with respect to x | |
| (i) $(2e^x - 3)^6$ | 2 |
| (ii) $\frac{x^2}{\sin x}$ | 2 |
| | |
| d) Integrate xe^{x^2} | 1 |

Question 14 – Start A New Booklet – (15 marks)

Marks



Ship A sails 20 nautical miles from Port P on a bearing of 035° . Ship B is 36 nautical miles from Port P on a bearing of 110° .

- (i) Copy the diagram into your answer booklet and mark on it all the given information. 1
 - (ii) Show $\angle APB = 75^\circ$ 1
 - (iii) Use the cosine rule to determine the distance between the two ships, to the nearest nautical mile. 2
- b) State the domain and range of $y = \sqrt{1-x}$ 2
- c) The gradient function of a curve is given by $f'(x) = 2(x-1)(x+4)$ and the curve passes through the point $(0, 8)$
- (i) Find the equation of $f(x)$ 2
 - (ii) Sketch the curve clearly labelling turning points and the y -intercept. 2
 - (iii) For what values of x is the curve concave up? 1
- d) Consider the function
- $$y = \ln(x-3) \quad x > 3$$
- (i) Sketch the function, showing its essential features. 2
 - (ii) Use Simpson's Rule with 3 function values to find an approximation to 2

$$\int_4^6 \ln(x-3) dx$$

Question 15 – Start A New Booklet – (15 marks)

Marks

- a) The first three terms of a GP are 0.1, 0.12, 0.144

(i) Find the 40th term, correct to 1 decimal place.

2

(ii) Calculate the sum of the first 40 terms, correct to 1 decimal place.

2

- b) A particle moves in a straight line so that its displacement, in metres, is given by:

$$x = \frac{t - 4}{t + 1}$$

where t is measured in seconds.

(i) What is the displacement when $t = 0$

1

(ii) Show that $x = 1 - \frac{5}{t+1}$, and hence find expressions for the velocity and acceleration in terms of t

3

(iii) Is the particle ever at rest? Give a reason for your answer.

1

- c) The population of a certain insect is growing exponentially according to

$$N = 400e^{kt}$$

where t is the time in weeks after the insects are first counted. At the end of five weeks the insect population has doubled.

(i) Calculate the exact value of k .

2

(ii) How many insects will there be after 8 weeks?

1

(iii) At what rate is the population increasing after 5 weeks.

1

- d) (i) Write down the discriminant of $2x^2 + 4x + k$

1

(ii) For what values of k does $2x^2 + 4x + k = 0$ have real roots.

1

Question 16 – Start A New Booklet – (15 marks)

Marks

- a) (i) Find the equations of the tangent and normal to the curve with equation

$$y = 4x^2(1 - x)$$

at the point $(1,0)$.

3

- (ii) The tangent and normal cut the y -axis at A and B respectively. If the point of intersection of the tangent and the normal is C find the area of $\triangle ABC$.

2

- b) (i) Sketch the graphs of $y = e^{-x}$, $y = x + 1$ and $x = 2$ on the same set of axes.

1

- (ii) Find, by integration, the area bounded by $y = e^{-x}$, $y = x + 1$ and $x = 2$ (leave your answer in exact form)

2

- c) Find:

(i) $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$

1

(ii) $\lim_{x \rightarrow 0} \frac{\sin x}{x}$

1

- d) A store offers a special deal where it will charge no interest on loans on purchases for the first year, and charge 1% per month on the balance owing each month thereafter. However, normal repayments must be made at the end of each month. Emily decides to buy a \$4000 television using the special deal.

She agrees to repay the loan over 24 equal monthly repayments of \$ M . Let \$ A_n be the amount owing at the end of the n th month.

- (i) Find an expression for A_1 and A_{12}

2

- (ii) Show $A_{15} = (4000 - 12M)(1.01)^3 - M(1 + 1.01 + 1.01^2)$

1

- (iii) Find the value of M

2

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln \left(x + \sqrt{x^2 - a^2} \right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln \left(x + \sqrt{x^2 + a^2} \right)$$

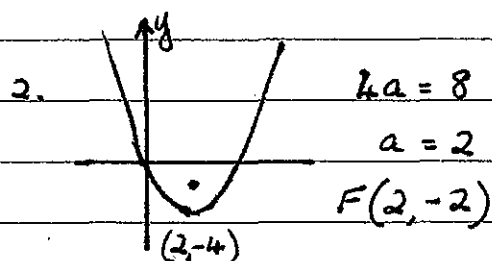
NOTE : $\ln x = \log_e x, \quad x > 0$

St. George Girls High School
2013 Trial Higher School Certificate Examination
Mathematics

Section 1 Multiple Choice

- | | |
|------|-------|
| 1. D | 6. D |
| 2. B | 7. A |
| 3. D | 8. D |
| 4. A | 9. A |
| 5. B | 10. D |

$$\begin{aligned}
 1. \quad 4x-1-kx-2 &= 6 \\
 -4+2k &= 6 \\
 2k &= 10 \\
 k &= 5
 \end{aligned}$$



$$\begin{aligned}
 3. \quad V &= \left(\frac{1}{2}\right)^3 \text{ of original} \\
 &= \frac{1}{8} \text{ of original} \\
 \therefore \text{ \# of statues} &= 8 \times 10 = 80
 \end{aligned}$$

$$\begin{aligned}
 4. \quad \angle DFE &= 180^\circ - 112^\circ \\
 &= 68^\circ \\
 \angle DEF &= 68^\circ \\
 \therefore x + 68 &= 112 \\
 x &= 44
 \end{aligned}$$

$$\begin{aligned}
 5. \quad y &= 0 \text{ when } x = 0, 1, 2 \\
 \text{When } x < 0 \quad y &> 0 \\
 \therefore B & \text{ (not A)}
 \end{aligned}$$

$$\begin{aligned}
 6. \quad \text{Amplitude } &4 \\
 \text{Period} &= \frac{2\pi}{\frac{1}{2}} \\
 &= 4\pi
 \end{aligned}$$

$$\begin{aligned}
 7. \quad \text{When } t=0 \quad x &= 8 \\
 8 &= 0 - 0 + c \\
 x &= \frac{1}{2}t^2 - 4t + 8 \\
 \dot{x} &= t - 4 \\
 \text{At rest when } t &= 4 \\
 x &= \frac{1}{2} \times 4^2 - 4 \times 4 + 8 \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 8. \quad \text{Negative gradient } f'(a) &< 0 \\
 \text{Concave up } f''(a) &> 0
 \end{aligned}$$

$$9. \text{ Test } (0,0) \Rightarrow A$$

$$\begin{aligned}
 10. \quad d &= \frac{|3 \times 2 - 1 - 4|}{\sqrt{3^2 + (-1)^2}} \\
 &= \frac{|6 - 1 - 4|}{\sqrt{10}} \\
 &= \frac{1}{\sqrt{10}}
 \end{aligned}$$

Question 11

$$(a) -2.560\overline{2295} \dots \\ = -2.56 (2 \text{ dp})$$

$$(f) \frac{125 - 8p^3}{(5-2p)(25+10p+4p^2)}$$

$$(b) |4-2x| \leq 6 \\ -6 \leq 4-2x \leq 6 \\ -10 \leq -2x \leq 2 \\ 5 \geq x \geq -1 \\ -1 \leq x \leq 5$$

$$(g) \text{ Primitive of } \frac{6-3x^2}{x^2} \\ = \int 6x^{-2} - 3 dx$$

$$= \frac{6x^{-1}}{-1} - 3x + c$$

$$(c) \frac{1}{2-\sqrt{3}} + \frac{1}{2\sqrt{2}+3}$$

$$= -\frac{6}{x} - 3x + c$$

$$= \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} + \frac{1}{2\sqrt{2}+3} \times \frac{2\sqrt{2}-3}{2\sqrt{2}-3}$$

$$= \frac{2+\sqrt{3}}{4-3} + \frac{2\sqrt{2}-3}{8-9}$$

$$= \frac{2+\sqrt{3}}{1} + \frac{2\sqrt{2}-3}{-1}$$

$$= 2+\sqrt{3} - 2\sqrt{2} + 3$$

$$= 5+\sqrt{3}-2\sqrt{2}$$

$$(d) \frac{d(x^3 \ln x)}{dx}$$

$$= 3x^2 \ln x + x^3 \cdot \frac{1}{x}$$

$$= 3x^2 \ln x + x^2$$

$$(e) \tan \theta = \sqrt{3} \quad 0 \leq \theta < 2\pi$$

$$\theta_{\text{acute}} = \frac{\pi}{3}$$

$$\theta = \frac{\pi}{3}, \pi + \frac{\pi}{3}$$

$$= \frac{\pi}{3}, \frac{4\pi}{3}$$

Question 2

(a) $e^{3.7}$

$$= 40.447304\dots$$

$$= 40.45 \text{ (4 sig figs)}$$

$$\therefore 5 + 9d = 4(5 + d)$$

$$= 20 + 4d$$

$$5d = 15$$

$$d = 3$$

(b) $A = \frac{1}{2}r^2\theta$

$$= \frac{1}{2} \times 16^2 \times \frac{\pi}{3}$$

$$= \frac{128\pi}{3}$$

$$\text{Area is } \frac{128\pi}{3} \text{ cm}^2$$

(f) $\angle ABE = 180^\circ - (85^\circ + 50^\circ)$

(angle sum of a triangle)

$$= 45^\circ$$

$$\therefore \angle ABE = \angle ECD$$

$$= 45^\circ$$

(c) $f(3) + f(0) - f(-2)$

$$= 3 \times 3 - 6 + (4 - 0^2) - (-2 + 2)$$

$$= 3 + 4 - 0$$

$$= 7$$

$\therefore AB \parallel CD$ (alternate angles are equal)

(g) $\frac{d}{dx} \log_e(\cos x) = \frac{1}{\cos x} \cdot -\sin x$

$$= -\tan x$$

(d) $3^{2x} - 6(3^x) - 27 = 0$

$$\text{Let } m = 3^x$$

$$m^2 - 6m - 27 = 0$$

$$(m - 9)(m + 3) = 0$$

$$m = 9, -3$$

$$3^x = 9 \text{ or } 3^x = -3$$

$$x = 2 \quad \text{no solution}$$

$$\text{since } 3^x > 0$$

$$\text{for all } x.$$

$$\therefore x = 2$$

$$\therefore \int \tan x \, dx = -\log_e(\cos x) + c$$

(e) Let d be the common diff

$$t_2 = a + d$$

$$= 5 + d$$

$$t_{10} = a + 9d$$

$$= 5 + 9d$$

Question 13

$$\begin{aligned} \text{(a)(i) Grad CA} &= \frac{4-0}{7-1} \\ &= \frac{4}{6} \\ &= \frac{2}{3} \end{aligned}$$

$$\begin{aligned} \text{(ii) } \tan \theta &= \frac{2}{3} \\ \theta &= 34^\circ \text{ (nearest degree)} \end{aligned}$$

$$\begin{aligned} \text{(iii) Equation of CA} \\ y - 0 &= \frac{2}{3}(x - 1) \\ y &= \frac{2}{3}x - \frac{2}{3} \end{aligned}$$

$$\text{(iv) Midpoint CA} = \left(\frac{1+7}{2}, \frac{0+4}{2} \right)$$

$$D = (4, 2)$$

$$\begin{aligned} \text{(v) Grad BD} &= \frac{8-2}{0-4} \\ &= \frac{6}{-4} \\ &= -\frac{3}{2} \end{aligned}$$

$$\text{Grad BD} \times \text{Grad CA} = \frac{2}{3}x - \frac{3}{2}$$

$$= -1$$

$$\therefore BD \perp CA$$

$$\begin{aligned} \text{(vi) AC} &= \sqrt{(7-1)^2 + (4-0)^2} \\ &= \sqrt{52} \end{aligned}$$

$$\begin{aligned} BD &= \sqrt{(0-4)^2 + (8-2)^2} \\ &= \sqrt{52} \end{aligned}$$

$$\begin{aligned} \text{Area } \triangle ABC &= \frac{1}{2} \times \sqrt{52} \times \sqrt{52} \\ &= 26 \text{ units}^2 \end{aligned}$$

$$\text{(c)(i) } \frac{d(2e^x - 3)^6}{dx}$$

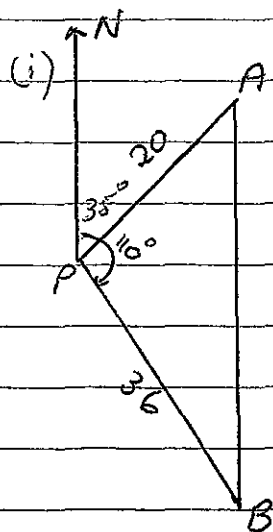
$$\begin{aligned} &= 6(2e^x - 3)^5 \cdot 2e^x \\ &= 12e^x (2e^x - 3)^5 \end{aligned}$$

$$\begin{aligned} \text{(ii) } \frac{d\left(\frac{x^2}{\sin x}\right)}{dx} &= \frac{2x \sin x - x^2 \cos x}{(\sin x)^2} \\ &= \frac{2x \sin x - x^2 \cos x}{\sin^2 x} \end{aligned}$$

$$\begin{aligned} \text{(b) LHS} &= 2\cos^2 \theta + 1 \\ &= 2(1 - \sin^2 \theta) + 1 \\ &= 2 - 2\sin^2 \theta + 1 \\ &= 3 - 2\sin^2 \theta \\ &= \text{RHS.} \end{aligned}$$

$$\begin{aligned} \text{(d) } \int x e^{x^2} dx &= \frac{1}{2} \int 2x e^{x^2} dx \\ &= \frac{1}{2} e^{x^2} + c \end{aligned}$$

Question 14



(ii) $\angle APB = 110^\circ - 35^\circ = 75^\circ$

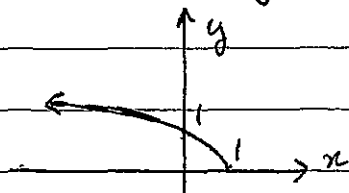
(iii) $AB^2 = 20^2 + 36^2 - 2 \times 20 \times 36 \cos 75^\circ$
 $= 1323.30...$
 $AB = 36.3771...$

$\therefore$ Ships are 36 nmiles (nearest nm) apart.

(b) $y = \sqrt{1-x}$

Domain: $1-x \geq 0$
 $x \leq 1$

Range: $y \geq 0$



(c) $f'(x) = 2(x-1)(x+4)$
 $= 2(x^2 + 3x - 4)$
 $= 2x^2 + 6x - 8$
 $f(x) = \frac{2x^3}{3} + \frac{6x^2}{2} - 8x + C$

$f(0) = 8 \therefore C = 8$

$f(x) = \frac{2}{3}x^3 + 3x^2 - 8x + 8$

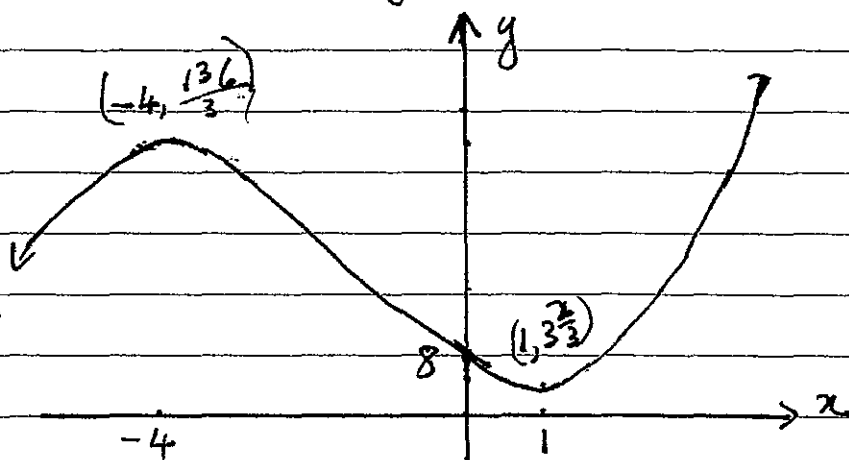
(ii) Stationary points occur when $f'(x) = 0$

ie $x = 1$ or $x = -4$

$y = 3\frac{2}{3}$

$y = \frac{136}{3} = 45\frac{1}{3}$

$f(0) = 8$ (y intercept)



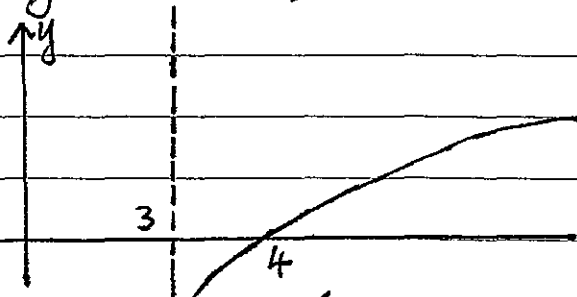
(iii) $f''(x) = 4x + 6$

Curve is concave up when $f''(x) > 0$

$4x + 6 > 0$

$x > -\frac{3}{2}$

(d) (i) $y = \ln(x-3)$ $x > 3$



(ii) $\int_4^6 \ln(x-3) dx \div \frac{1}{3} (y_1 + 4y_2 + y_3)$

$h = 1$

$y_1 = \ln(4-3) = 0$

$y_2 = \ln(5-3) = \ln 2$

$y_3 = \ln(6-3) = \ln 3$

$= \frac{1}{3} (0 + 4\ln 2 + \ln 3)$

$= \frac{1}{3} \ln 48$

Question 15

(a) $a = 0.1$ $r = 1.2$

(iii) $\frac{5}{(t+1)^2} > 0$ for all values of t
and hence $v \neq 0$, ie particle
is never at rest.

$$\begin{aligned} \text{(i)} \quad t_{40} &= ar^{39} \\ &= 0.1 \times 1.2^{39} \\ &= 122.480... \\ &= 122.5 \text{ (1dp)} \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad N &= 400e^{kt} \\ \text{(i)} \quad \text{When } t=0 \quad N &= 400e^0 = 400 \\ \text{When } t=5 \quad N &= 2 \times 400 = 800 \\ \therefore 800 &= 400e^{5k} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad S_{40} &= \frac{a(r^{40}-1)}{r-1} \\ &= \frac{0.1(1.2^{40}-1)}{1.2-1} \\ &= 734.3875... \\ &= 734.4 \text{ (1dp)} \end{aligned}$$

$$\begin{aligned} e^{5k} &= 2 \\ 5k &= \log_e 2 \\ k &= \frac{\log_e 2}{5} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \text{(i)} \quad x &= \frac{t-4}{t+1} \\ \text{When } t=0 \quad x &= \frac{-4}{1} \\ \text{Displacement is } &-4\text{m} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad \text{When } t=8 \quad N &= 400e^{8k} \\ &= 400e^{\frac{8 \log_e 2}{5}} \\ &= 1212.57... \\ \therefore \text{There will be } &1213 \text{ insects after 8 weeks} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad 1 - \frac{5}{t+1} &= \frac{t+1-5}{t+1} \\ &= \frac{t-4}{t+1} \\ \therefore x &= 1 - \frac{5}{t+1} \\ \text{OR } \frac{t-4}{t+1} &= \frac{t+1-5}{t+1} \\ &= \frac{t+1-5}{t+1} \\ &= 1 - \frac{5}{t+1} \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad \frac{dN}{dt} &= 400k e^{kt} \\ &= kN \\ \text{When } t=5 \quad N &= 800 \\ \frac{dN}{dt} &= \frac{\log_e 2}{5} \times 800 \\ &= 110.903... \end{aligned}$$

Population is increasing at a rate
of 111 insects/week.

$$\begin{aligned} v = \dot{x} &= \frac{d}{dt}(1 - 5(t+1)^{-1}) \\ &= +5(t+1)^{-2} \times 1 \\ &= \frac{5}{(t+1)^2} \end{aligned}$$

$$\text{(d)} \quad \text{(i)} \quad \Delta = 4^2 - 4 \times 2 \times k = 16 - 8k$$

$$\begin{aligned} a = \ddot{x} &= \frac{d}{dt} 5(t+1)^{-2} \\ &= -10(t+1)^{-3} \cdot 1 \\ &= \frac{-10}{(t+1)^3} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad 2x^2 + 4x + k &= 0 \text{ has real} \\ \text{roots when } \Delta &\geq 0 \\ 16 - 8k &\geq 0 \\ 8k &\leq 16 \\ k &\leq 2 \end{aligned}$$

Question 16

(a) (i) $y = 4x^2(1-x)$
 $= 4x^2 - 4x^3$

$$\frac{dy}{dx} = 8x - 12x^2$$

When $x = 1$

$$\frac{dy}{dx} = 8 - 12$$

$$\frac{dy}{dx} = -4$$

$\therefore$ Grad of tangent $= -4$
 at $(1, 0)$

Grad of normal $= \frac{1}{4}$
 at $(1, 0)$

Eqⁿ of tangent is

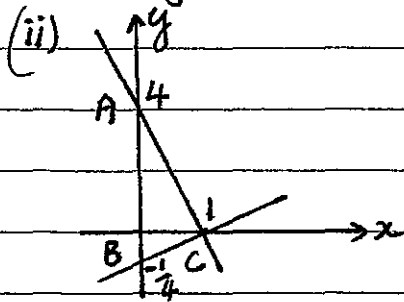
$$y - 0 = -4(x - 1)$$

$$y = -4x + 4$$

Eqⁿ of normal is

$$y - 0 = \frac{1}{4}(x - 1)$$

$$y = \frac{1}{4}x - \frac{1}{4}$$



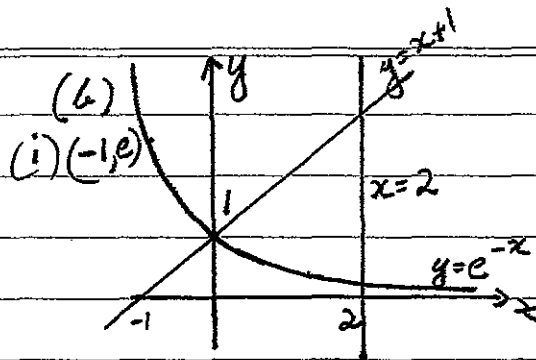
$$AB = 4\frac{1}{4} = \frac{17}{4}$$

$$OC = 1$$

$$\text{Area } \triangle ABC = \frac{1}{2} \times OC \times AB$$

$$= \frac{1}{2} \times 1 \times \frac{17}{4}$$

$$= \frac{17}{8} \text{ units}^2$$



(ii) $A = \int_0^2 x + 1 - e^{-x} dx$

$$= \left[\frac{x^2}{2} + x + e^{-x} \right]_0^2$$

$$= \left(\frac{4}{2} + 2 + e^{-2} \right) - (0 + 0 + e^0)$$

$$= 0 + 0 + 3 + e^{-2}$$

$$\text{Area} = 3 + \frac{1}{e^2} \text{ units}^2$$

(c) (i) $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3} = \lim_{x \rightarrow 3} \frac{(x+3)(x-3)}{(x-3)}$

$$= \lim_{x \rightarrow 3} (x+3)$$

$$= 6$$

(ii) $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

(d) (i) $A_1 = 4000 - M$

$$A_2 = A_1 - M = 4000 - 2M$$

$$A_3 = A_2 - M = 4000 - 3M$$

By the same pattern

$$A_{12} = 4000 - 12M$$

(ii) $A_{13} = A_{12} + \text{interest} - M$
 $= A_{12} \times 1.01 - M$

$$= 4000 \times 1.01 - 12M \times 1.01 - M$$

$$A_{14} = A_{13} \times 1.01 - M$$

$$= 4000 \times 1.01^2 - 12M \times 1.01^2 - M \times 1.01 - M$$

$$A_{15} = A_{14} \times 1.01 - M$$

$$= 4000 \times 1.01^3 - 12M \times 1.01^3 - M \times 1.01^2 - M \times 1.01 - M$$

(iii) By the same pattern

$$\begin{aligned}A_{24} &= (4000 - 12M) \times 1.01^{12} - M \times 1.01^{11} - M \times 1.01^{10} - \dots - M \\&= (4000 - 12M) \times 1.01^{12} - M(1 + 1.01 + 1.01^2 + \dots + 1.01^{11}) \\&= (4000 - 12M) \times 1.01^{12} - \frac{M \cdot 1(1.01^{12} - 1)}{1.01 - 1}\end{aligned}$$

If loan repaid after 24 months then $A_{24} = 0$

$$0 = (4000 - 12M) \times 1.01^{12} - \frac{M(1.01^{12} - 1)}{0.01}$$

$$100M(1.01^{12} - 1) = (4000 - 12M) \times 1.01^{12}$$

$$100M(1.01^{12} - 1) + 12M \times 1.01^{12} = 4000 \times 1.01^{12}$$

$$M[100(1.01^{12} - 1) + 12 \times 1.01^{12}] = 4000 \times 1.01^{12}$$

$$M = \frac{4000 \times 1.01^{12}}{112 \times 1.01^{12} - 100}$$

$$= 172.00544 \dots$$

Repayments are \$172 per month